Travelling Salseman Problem

Travelling Salseman Problem

- In travelling salseman problem a salse person visit each & every city exactly once& return to original city.
- A tour to be simple path that starts & ends at vertex 1.
- Every tour consists of an edge(1,k) for some k € V-{1} and a path from vertex k to vertex 1.
- The path from vertex k to vertex in v-{1,k} exactly once.
- It is easy to see that if the tour is optimal , then the path from k to 1 must be a short test k to 1 path going through all vertices in V-{1,k}.

TSP(Continues..)

- Let g(i,s) be the length of a shortest path starting at vertex I, going through all vertices all vertices in s and terminates at vertex 1.
- The function g(1,v-[1]) is the length at vertex I, going through all vertices in s and terminating at vertex 1.
- The function g(1,v-{1}) is the length of an optimal salseperson tour.

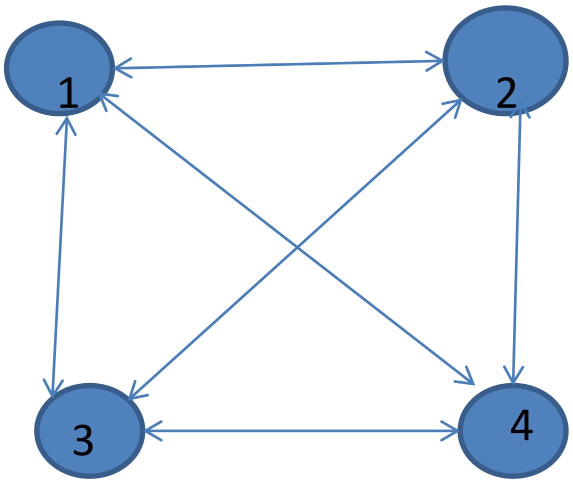
TSP

- This problem can be formulated as
- $g(1, v-\{1\}) = \min\{c1k + g(k, v-\{1,k\})\}$…………….(1)
- (Note minimum is $2 <= k <= n$)
- Generalizing 1, we obtain
- $G(i, s) = \min\{c_{ij} + g(j, s-\{j\})\}$…………………………..(2)
- (Note: minimum is j€S

TSP

- Example:

$$\begin{bmatrix} 0 & 5 & 7 & 8 \\ 2 & 0 & 3 & 5 \\ 8 & 9 & 0 & 10 \\ 12 & 13 & 11 & 0 \end{bmatrix}$$



- To calculate g(1,[2,3,4]) = ? We apply the formula
- g(1,v-{1})=min{c1k+g(k, v-{1,k})}................(1)
- g(i, s)=min j€s{c_{ij}+g(j,s-{j})}.........................(2)
- Updating eq.(2) we get
- $g(i,\phi)=C_{i1}, 1<=i<=n$
- Thus

$g(2,\phi)=C_{21}=2$

$g(3, \phi)=C_{31}=8$

$g(4, \phi)=C_{41}=12$

- $g(2,\{3\})=min\{C_{23}+g(3,\{3\}-\{3\})\}$
- (Note min is 3€{3})
- $g(2,\{3\})=min\{C_{23}+g(3,\phi)\}$

  $=C_{23}+g(3,\phi)$

  $=C_{23}+C31$

  $=3+8=11$

  $g(2,\{4\})=min\{C_{24}+g(4,\{4\}-\{4\})\}$
- (Note min is 3€{3})
- $g(2,\{3\})=min\{C_{24}+g(4,\phi)\}$

  $=C_{24}+g(4,\phi)$

  $=C_{24}+C_{41}$

  $=5+12=17$

Similarly,

$g(3,\{2\})=C_{32}+g(2,\phi)$

$=C_{32}+C_{21}=9+2=11$

$g(3,\{4\})=C_{34}+g(4,\phi)$

$=C_{34}+C_{41}=10+12=22$

$g(4,\{2\})=C_{42}+g(2,\phi)$

$=C_{42}+C_{21}=13+2=15$

$g(4,\{3\})=C_{41}+g(3,\phi)$

$=C_{43}+C_{31}=11+8=19$

Next we compute g(i,s) with |s|=2,i≠1,

g(i, s)=min j€s $\{c_{ij}+g(j,s-\{j\})\}$

$g(2,\{3,4\})=\min\{C_{23}+g(3,\{3,4]-\{3\},C_{24}+g(4,\{3,4\}-\{4\})\}$

$=\min\{C_{23}+g(3,\{4\}),C_{24}+g(4,\{3\})\}$

$=\min\{3+22,5+19\}=\min\{26,24\}=24$

$g(3,\{2,4\})=\min\{C_{32}+g(2,\{2,4]-\{2\},C_{34}+g(4,\{2,4\}-\{4\})\}$

$=\min\{C_{32}+g(2,\{4\}),C_{34}+g(4,\{2\})\}$

$=\min\{9+17,10=15\}=\min\{26,25\}=25$

$g(4,\{2,3\})=\min\{C_{42}+g(2,\{3\}),C_{43}+g(3,\{2\})\}$

$=\min\{13+11,11+11\}=\min\{24,22\}=22$

Finally from(1) we obtain

g(1,{2,3,4})=min{C12+ g(2,{3,4}),C13+g(3,{2,4}),C14+ g(4,{2,3})}

=MIN{29,32,30}=29

An optimal tour for given graph is 29

Application

- Computer wiring
- Vehicle routing
- Clustering

Scope of research

- Polynomial Time Solvable Variations

Assignment

- Q.1)What is Travelling Salseman problem?
- Q.2)Explain travelling Salseman problem with example